102-05

Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016.

Second Semester

Civil Engineering

MA 1151 — MATHEMATICS – II

(Common to all Branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of $\frac{t}{e^t}$

2. Verify initial value theorem or the function $f(t) = ae^{-bt}$.

3. Find a, b, c such that $\vec{F} = (3x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (3x + cy + 3z)\vec{k}$ is irrotational.

4. State Green's theorem in a plane.

5. If $f(z)$ is an analytic function whose real part is constant, prove that $f(z)$ is a constant function.

6. Find the invariant points of the transformation $w = \frac{z-1}{z+1}$.

7. Sketch the region of integration of the integral $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} y\,dx\,dy$.

8. Evaluate $\int_0^1 \int_0^z \int_0^{y+z} dx\ dy\ dz$.

9. Define singular point.

10. Find the residue of $f(z) = \tan z$ at its poles.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Laplace transform of the following functions

(1) $t^2 e^{-t} \cos t$

(2) $\dfrac{e^{at} - \cos bt}{t}$. (8)

(ii) Using convolution theorem, find $L^{-1}\left\{\dfrac{1}{(s^2+a^2)^2}\right\}$. (8)

Or

(b) (i) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ $f(t+2a) = f(t)$. (8)

(ii) Using Laplace transform, solve $\dfrac{d^2y}{dt^2} + y = \sin 2t$, with $y(0) = 0$ and $y'(0) = 0$. (8)

12. (a) (i) Find the angle between the surfaces $x \log z = y^2 - 1$ and $x^2 y = 2 - z$ at the point $(1,1,1)$. (8)

(ii) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. (8)

Or

(b) Verify divergence theorem for the function $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ taken over the cube bounded by the planes $x = 0, x = 1$, $y = 0, y = 1$, $z = 0$, $z = 1$.

13. (a) (i) If $f(z) = u + iv$ is an analytic function of prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)(\log|f(z)|) = 0$. (8)

(ii) Find the image of the square whose vertices are $z = 1 + i$, $3 + i$, $1 + 3i$ and $3 + 3i$ under the transformation $w = \dfrac{1}{z}$. (8)

Or

(b) (i) Determine the analytic function $f(z) = u + iv$, if $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

(ii) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $W = i, 0, \infty$. (8)

14. (a) (i) Evaluate the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\, dxdy$ by charging to polar coordinates.

(ii) Evaluate $\int_0^1 \int_{4y}^4 e^{x^2} dxdy$ by changing the order of integration. (8)

Or

(b) (i) Evaluate $\iint x^2y^2\, dx\, dy$ over the circle $x^2 + y^2 = 1$. (8)

(ii) Prove that the volume enclosed by the cylinder $x^2 + y^2 = 2ax$ and $z^2 = 2ax$ is $\frac{128a^3}{15}$. (8)

15. (a) (i) Determine the Laurent's series expansion of $f(z) = \frac{7z-2}{(z+1)z\,(z-2)}$ in the region $1 < |z+1| < 3$. (8)

(ii) By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$. (8)

Or

(b) (i) Using Cauchy's integral formula evaluate $\int_C \frac{e^z}{\left(z^2+\pi^2\right)^2} dz$ where C is the circle $|z| = 4$. (8)

(ii) Evaluating using contour integration $\int_0^\infty \frac{\cos ax}{x^2+1} dx,\ a \geq 0$. (8)